# Gradient-Enhanced Plasticity Theory for Size Effects and Mesh-Independent Numerical Results

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**Abstract:** In this work, a thermodynamically consistent constitutive formulation for the coupled thermomechanical strain gradient plasticity theory is developed in the context of the finite deformation framework. A corresponding finite element solution is presented to investigate the micro-structural features of metallic volumes. Size effects and mesh-sensitivity are simulated using the developed numerical framework to test the feasibility of the proposed gradient-enhanced plasticity theory.

**Keywords:** strain gradient plasticity; size effects; mesh-sensitivity; finite element method.

## 1 Introduction

In the authors' previous works [1-9], the coupled thermo-mechanical strain gradient plasticity model was proposed to investigate the microstructural features of metallic materials and solved numerically in the context of one-dimensional small deformation finite element implementation. In the current work, a two-dimensional finite element solution for finite deformation considering the temperature effect is proposed. The developed model is established based on an extra Helmholtz-type partial differential equation, and the nonlocal quantity is calculated in a coupled method based on the equilibrium conditions. This approach is well known for its computational strength, however, it is also commonly accepted that it cannot capture the size effect phenomenon observed in the micro-/nanoscale experiments during hardening. In order to resolve this issue, a modified strain gradient approach which can capture the size effects under the finite deformation is constructed in this work. The simple shear problem is then solved to carry out the feasibility study of the developed model on the size effect phenomenon. Lastly, the uniaxial plane strain tension problem with shear bands is examined to perform the mesh sensitivity tests of the model during softening.

## 2 Implicit Gradient-enhanced Plasticity Theory

The implicit gradient-enhanced plasticity theory (IGT) is introduced as an alternative method with only a $C^0$-continuous requirement to overcome a computational inherent difficulty in the implicit gradient-enhanced plasticity theory. In the IGT, the Helmholtz-type partial differential equation (PDE) is obtained, and the nonlocal quantities are calculated in a coupled method based on the equilibrium conditions. The nonlocal effective plastic strain $\bar{\varepsilon}^p$, which indicates a weighted average of the local effective plastic strain $\varepsilon^p$, is introduced and obtained by the following PDE with the material length scale parameter $l_p$:

$$\bar{\varepsilon}^p - l_p^2 \nabla^2 \bar{\varepsilon}^p = \varepsilon^p \tag{1}$$

## 3 Result and discussion

### Size effects

The schematic illustration of an infinitely long plate geometry, the initial conditions, and the macroscopic and microscopic boundary conditions under shear is shown in Figure 1. The direct comparison between the non-gradient plasticity theory and the proposed IGT is given in Figure 2 on the size effects in terms of the true stress versus true strain behaviors.

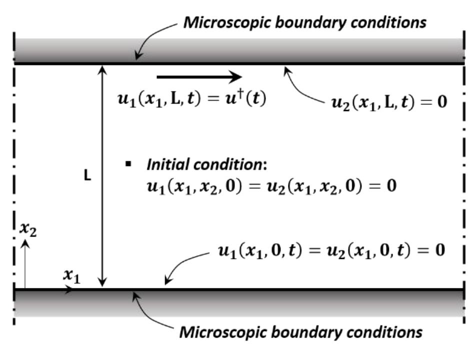


**Fig.1 Shearing of a layer problem: macro-/microscopic boundary conditions, initial conditions, and geometry.**

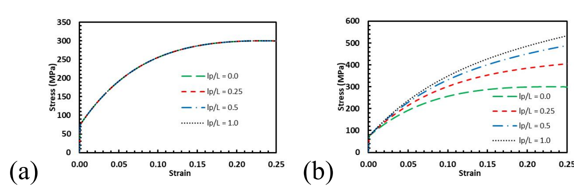


**Fig.2 Direct comparison between (a) non-gradient plasticity theory and (b) the proposed IGT on the size effects.**

## 4 Mesh-sensitivity tests

The schematic illustration of a plane strain problem under uniaxial tensile loading to explore the mesh-sensitivity characteristics of the proposed model during strain softening is presented in Figure 3. To do this, the three different mesh sizes, i.e. Mesh 1: 10×50=500, Mesh 2: 15×75=1125, and Mesh 3: 20×100=2000 elements are used. Figure 4 shows the comparison between non-gradient plasticity theory and the proposed IGT. As expected, the non-gradient theory results in strong mesh-dependent behaviors, whereas, the proposed IGT results in mesh-insensitive results.

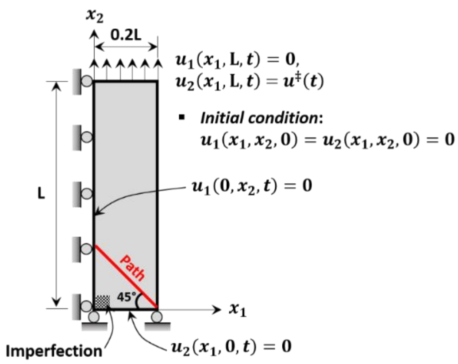


**Fig.3 Plane strain problem under uniaxial tensile loading: boundary conditions, initial conditions, and geometry.**

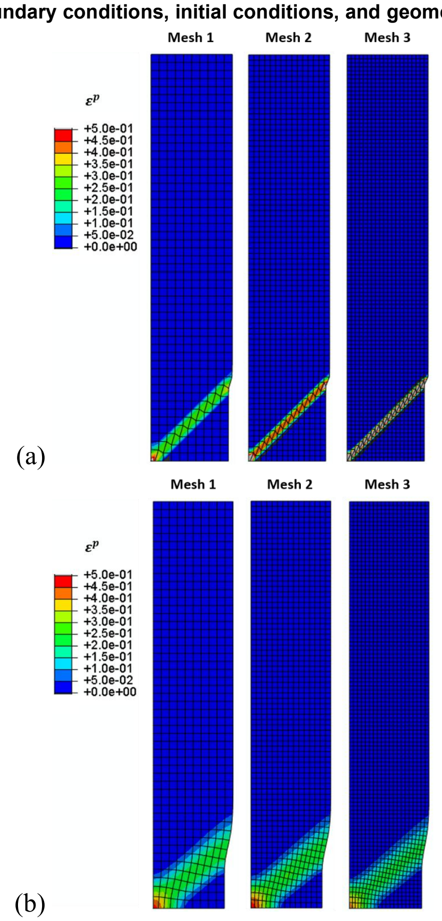


**Fig.4 Direct comparison between (a) non-gradient plasticity theory and (b) the proposed IGT on the mesh-sensitivity tests.**

## 5 Conclusion

The thermodynamically consistent constitutive model for the coupled thermo-mechanical strain gradient plasticity theory is constructed in the finite deformation context. In the IGT, it was well observed that the material responses during hardening were stiffer as the sizes of the sample got smaller, viz. the size effect. The main cause of this difference was the existence of the nonlocal effective plastic strain in the yield criterion. In addition, the mesh-sensitivity of the proposed model on the strain localization during strain softening was examined in this work. In the classical non-gradient plasticity model with no gradient regularization, it was well observed that the widths of the shear band drastically vary according to the mesh size, whereas the mesh-independent material responses are obtained in the proposed model.

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