Numerical Simulation of Multi-Mathematical Model Coupling in the Squeeze Casting Process

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Abstract: This paper presents the development of an SPH-FEM coupled computational model, combining the Smoothed Particle Hydrodynamics (SPH) method with the Finite Element Method (FEM), for numerical simulation of the squeeze casting process. Drawing upon the unique strengths of the SPH method in simulating free surfaces, a computational framework for numerical simulation of the filling process in squeeze casting is established, maximizing its capabilities in modeling flow fields. Furthermore, the temperature field results are transmitted as initial data to the finite element model, thereby achieving the coupling between SPH and FEM. This integration leverages the advantages of the FEM in addressing stress-strain problems. Consequently, a finite element model based on the FEM method is constructed to conduct numerical simulation of the solidification process in squeeze casting, enabling the analysis of its thermal stress field.

Keywords: squeeze casting; smoothed particle hydrodynamics; finite element method; coupled model

1 Introduction

Squeeze casting is an advanced metal forming process that combines casting and forging. It produces high-performance castings with few defects by solidifying the molten metal in the cavity under high pressure [1]. However, due to limitations in domestic equipment and process levels, defect issues are difficult to avoid in actual production. Simulation technology can provide solutions covering the entire product lifecycle for the squeeze casting process, thereby improving production efficiency and shortening the production cycle [2].

Smoothed Particle Hydrodynamics (SPH) is a meshless numerical method used to simulate fluid motion. It discretizes continuous media into a series of interacting particles to reflect the changing behaviors of mass, energy, and momentum [3]. Due to the unique advantages of the SPH method in calculating free surfaces, a mathematical model is established using the SPH method to simulate the filling process of squeeze casting.

The Finite Element Method (FEM) is a numerical computation technique for finding approximate solutions to boundary value problems involving partial differential equations [4]. It divides the complex continuum into a finite number of non-overlapping units, lists the equations and solves them. Given the advantages of FEM in calculating stress-strain problems, this paper adopts the Finite Element Method to establish a mathematical model for simulating and calculating the solidification process of squeeze casting.

2 Mathematical modelling and numerical methods

Mathematical modelling of the filling process

The particle approximation for the field function and its spatial derivative at particle i is given as follows:

$$\langle f(x_i)\rangle = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x_j) W_{ij}$$

$$\langle \nabla \cdot f(x_i)\rangle = \sum_{j=1}^{N} \frac{m_j}{\rho_j} f(x_j) \cdot \nabla_i W_{ij}$$

where, i denotes the central particle, j represents particles within the support domain, $<f(x_i)>$ is the approximate function for particle i, N is the total number of particles j, W_{ij} is the smoothing kernel function, ∇ is the gradient operator, The kernel function is chosen to be a cubic spline function.

By employing the SPH particle approximation method, the Navier-Stokes equations are discretized to obtain the SPH expressions for the density equation, momentum equation, and energy equation. These equations are presented as follows:

$$\frac{\mathrm{d}\rho_i}{\mathrm{d}t} = \sum_{j=1}^{N} m_j v_{ij}^{\beta} \frac{\partial W_{ij}}{\partial x_i^{\beta}}$$

$$\frac{\mathrm{d}v_i^{\alpha}}{\mathrm{d}t} = \sum_{j=1}^{N} m_j \left(\frac{p_i}{\rho_i^2} + \frac{p_j}{\rho_j^2}\right) \frac{\partial W_{ij}}{\partial x_i^{\beta}}$$

$$\frac{\mathrm{d}e_i}{\mathrm{d}t} = \frac{1}{2} \sum_{j=1}^{N} m_j \frac{p_i + p_j}{\rho_i \rho_j} \cdot \frac{\partial W_{ij}}{\partial x_i^{\beta}}$$

where, ρ represents the density, v denotes the velocity, e is the internal energy of the fluid parcel, p is the pressure, α and β indicate coordinate directions.

Mathematical modelling of the solidification process

The equation of motion for three-dimensional finite element dynamics problems can be expressed as:

$$M\ddot{a}(t) + C\dot{a}(t) + Ka(t) = Q(t)$$

where, $a(t)$ represents the nodal displacement, $\dot{a}(t)$ is the nodal velocity, $\ddot{a}(t)$ is the nodal acceleration, and M, C, K, Q are respectively the mass matrix, damping matrix, stiffness matrix, and load vector.

During the solidification process in squeeze casting, the mechanical behavior of the metal within the mold cavity is highly complex. To address this, this paper employs the thermoelastic-viscoplastic constitutive model established by Zhu Wei [5] et al. for analysis:

$$\{\Delta\varepsilon\} = \{\Delta\varepsilon_{el}\} + \{\Delta\varepsilon_{th}\} + \{\Delta\varepsilon_{in}\}$$

where, $\{\Delta\varepsilon_{el}\}$ represents the increment of elastic strain, $\{\Delta\varepsilon_{th}\}$ denotes the increment of thermal strain, and $\{\Delta\varepsilon_{in}\}$ is the increment of inelastic strain.

3 Numerical simulation results

Filling process

Figure 1 shows the filling status of the molten metal at different time points, with the time taken for the molten metal to completely fill the cavity being 0.83 seconds. As can be seen from the figure, using the vertical counter-gravity squeeze casting process, the molten metal fills the cavity from bottom to top at a low speed, resulting in a smooth filling process that generally aligns with real-world conditions[6].

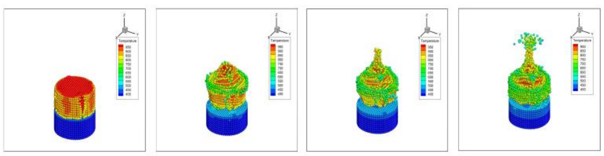

Figure1 The filling status and temperature distribution of the casting at different time points during the filling process: (a) *t*=0.03 s; (b) *t*=0.23 s; (c) *t*=0.33 s; (d) *t*=0.53 s

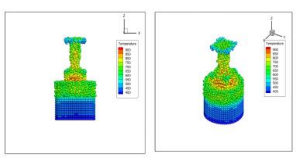

Figure 2 The filling status and temperature distribution of the casting at the end of the filling process

Solidification process

The simulation results of the temperature field during the solidification process of the casting are shown in Figure 3. The time required for the casting to solidify completely is 0.48 s, and the overall solidification order is from outside to inside.

Figure 4 presents the contour plot of equivalent stress during the solidification process of the casting. It can be observed from the figure that at the initial stage of solidification, the overall equivalent stress of the casting is relatively low. As the pressure from the punch continues to be applied, the overall equivalent stress of the casting gradually increases. The maximum equivalent stress is located at the thin-walled sections of the casting, and the stress distribution generally aligns with real-world conditions[3].

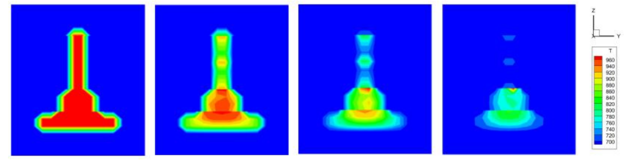

Fig. 3. The temperature field of the casting at different time points during the solidification process: (a) t=0s; (b) t=0.04s; (c) t=0.16s; (d) t=0.44s

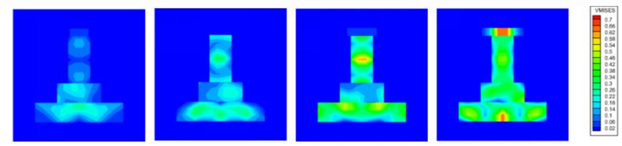

Figure 4 Contour plots of equivalent stress at different time points during the solidification process of the casting: (a) *t*=0.06 s; (b) *t*=0.19 s; (c) *t*=0.36 s; (d) *t*=0.40 s

4 Conclusion

This paper establishes a mathematical model for numerical simulation of the squeeze casting process based on the SPH-FEM coupling method. By analyzing the flow field, temperature field, and stress field during the filling and solidification processes of the I-shaped specimen in squeeze casting, the accuracy of the mathematical model established in this paper is verified, which has certain guiding significance for the actual production process of squeeze casting.

Acknowledgments

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